TITLE : **GIFTED CHILDREN'S INDUCTION OF STRATEGIES: METACOGNITIVE AND COGNITIVE STRATEGIES TO SOLVE MATHS AND CONVERSION PROBLEMS**

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ABSTRACT

Many authors, as Sternberg (1986), agree that cognitive abilities are an essential feature of giftedness. The higher extraordinary processes which regulate the task analysis and the self-control of each one's behaviour for solving problems, are considered an important component to identify the gifted among the average students.

Some of the gifted children's most important features are metacognition, speed to information processing, specially all that regards to automatic processing. Besides, the gifted start to use, spontaneously, effective strategies at an early age.

The present research will prove that they use their metacognitive capacity for solving maths problems from being six. Because they not only know which kind of processing they did to solve the problems arisen, but also they are capable of telling the strategies they used. All these points implies analysis and deduction capacity.

For these reasons, we were interested in studying their capacity for solving very complicated problems. We chose, as many well-known researchers (Simon, 1975; Anzai & Simon, 1979; Kadat, 1982), the famous Hanoi's tower Problem. This very hard to solve activity, specially with 5-6 rings, involves perceptive operations, complicated strategies to make the movements sequential and the memory operative.

INTRODUCTION

For a long time, the differential psychologists approached the matter of intelligence as other proffesionals did with electricity, or rather, they measured it without understanding its nature, despite they proposed a set of principles necessary to design the instruments for measuring.

A Sternberg & Berg's presentation (1986) dealt with the attribute sequences which the experts had used to define intelligence from 1921, and the important thing to emphasize regardless the time, is that the "higher level components" (abstract reasoning, representation, problem solving, taking decisions) are key aspects of intelligence (Colom, 1995, p. 95-98).

Simon (1978) considered that an individual faces a problem in the moment when he/she agrees to do a task ignoring how to solve it. Problem solving means all tasks demanding relatively complex procedures of reasoning, rather than a simple associative and routine process. Bourne et al. (1979) recognized three steps for solving problems : preparation, production and judgement (Vega, 1985,p. 494-5).

Generating "insight" processes is required both to understand a novel situation or task and to know how the individual concerned will face it. Such processes are measured by verbal, numerical, mistery and syllogism problems. All these activities imply logical reasoning, inference of relations and the reflexive use of the following processes : encoding, connection and selective comparison (Sternberg, 1981, p. 3-16).

An ample encoding task might be particularly important to make the problem solving efficient. Reasoning, problem solving and intelligence are so closely linked that any effort made to separate each is unlikely to succeed. Regardless what intelligence mean, it has been traditionally believed that some of its constituent sub-elements were **reasoning and problem solving.** Furthermore, both aspects would be comprised within whatever definition of intelligence (Sternberg, 1987, p. 361).

A · METACOGNITION AND GOGNITIVE STRATEGIES TO SOLVE MATHS PROBLEMS.

Children's observation when being tested by the MSCA, WPSSI, WISC-R and Stanford-Binet (Terman Merril, L-M form) intelligence psychometrical measurements, made the present research on **metacognitive capacity and strategies of problem solving** actual.

Certain factors such as memory capacity, linguistic comprehension and the skill to choose the representaion suitable for each problem have an influence on children's use of logic reasoning abilities, and all of them are different from the comprehension of logic by itself (Siegler & Richards, 1989, p. 1465).

The observation when solving "complex" problems as **Hanoi's Tower or the Problem of Radiations**, was achieved like playing a game during the prelimary interview before we proceeded to a child's testing or in the area of improving creativity or problem solving developed by the Extracurricular Enrichment programm (Psychopedagogical and Social Enrichment Model, MEPS). At this point, it must be underlined that cognitive abilities of problem solving are essential to succeed in any scientific or professional field.

Many authors, as Sternberg, agree that giftedness is built on cognitive abilities. Therefore, researchers believe that the gifted are outlined against average by their higher processes controling task processing and self-behaviour for solving problems. So, the metacognition and speed of information processing are attributed to gifted pupils.

"Metacognition" was named by Flavell (1975) and he suggested it was an individual's knowledge and awareness on his/her own cognitive processes.

Such high level strategies imply one's operational abilities dominion. According Campione, Brown & Ferrera (1982), this kind of meta-abilities usually arises massively at adolescence, but it may occur before (Benito, 1992, p. 72).

Being very interested in the metacognitive processes for solving maths problems, we proved that the gifted can use them from 6 years. To solve multiplication and division problems, they operate adding up, for example :

"if I've got 12 biscuits and want to share them with 4 children giving each equally, how many should I give each?"

> **Julián**, 6 years and 3 months, gave the right answer in few seconds, and he explained his mental activity that way: "adding up 3 + 3 + 3 + 3". Being asked why he didn't add 2 + 2 + 2 + 2, he replied that he did, but it was a wrong way. And we continued asking why not adding 1 + 1 + 1 + 1? and he answered "I already knew that added up 4".

Manuel, 6 years and 5 months, was proposed the following problem :

"if a bricklayer earned 36 pesetas and he was paid 4 pesetas/an hour, how many hours did he worked?"

> To solve it, he added 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 up to 36 and then, counted his fingers.

Carmen 7 years and 7 months (M.A.=13 years, QI= 172), showed her brilliant and outstanding capacity above all the children measured; she was asked this problem :

"if you have to pay 5 pesetas for 3 sweets, how much do 24 sweets cost?"

> In 7 seconds, she answered right. She explained she had added 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24; 8 times number 5 in 5: 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5= 40. But the more surprising thing was the symbolic images she did to solve the problems:

"if 3 metres of material costs 15 pesetas, how much will it cost 7 metres?"

> Carmen gave the right answer in 25 seconds; see attached photocopy.

"A jumper is usually sold by 32 pesetas. It was knocked 1/4 off its price. As nobody bought it, the owner gave again a discount at half the reduced cost, how much was the jumper finally sold by?"

> **Vicente**, 7 years and a month (M.A.= 13 years, IQ=199) answered the following in few seconds : 16 is half 32
> 8 is half 16
> they reduced 8 pesetas
> 32 - 8 = 24
> 12 is half 24.

"I put a plant of 8 centimetres high. One year later, it was 12 c high, another year later it was 18c high and when being 3 years it was 27c high; how high would it be after its four year long?"

Roberto, 8 years and 5 months (MA= 17 years; IQ = 211) answered in 7 seconds; and he explained to me the following : the difference among numbers is 1,5c each:

8 x 1,5 = 12; 12 x 1,5 = 18; 18 x 1,5 = 27; 27 x 1,5 = 40,5.

Therefore, metacognitive processes for solving maths problems seem to be developed at an early age, in contrast to children of average intelligence.

Gifted children are aware of their cognitive processes at an early age. So that, the acquisition of new knowledge and abilities is easier for them. In this sense, their speed of answering and capacity of managing un-taught maths concepts are really surprising; and, as it was previously mentioned, an average individual's dominion of higher level strategies occurs in his/her adolescence.

According to Benito (1994, p. 105), regular intelligence children are unable to explain how they solved a problem, and usually explain their mental activity in this terms : "it emerged that way", "whereas being sat in the bus, I put something here (pointing at his forehead) and then I learn everything"... These fellows refuse to approach any problem which seems hard to solve, arguing they have not been taught to multiply or divide. On the contrary, the gifted try to solve it regardless having been taught or knowing the matter.

We couldn't prove the existence of metacognitive capacity for solving problems before 5 years and 5 months all the research through, but a large automation of solving, speed and efficiency processes at this early age, even the problems dealing with division notion.

Pablo, 5 years and 7 months (mental age=8 years, IQ=151) was proposed this problem:

"if 12 biscuits are equally shared out among 4 children, how many are given each?"

No time was given to start the chronometer, he answered right automatically.

It is really remarkable that they are able to go beyond the **simple metacognive capacity** : they know not only which processes they followed to solve the problem, but also the strategies used, and this implies analysis and deduction capacity :

a - they are aware of knowing certain operations and able to use them automatically.

For instance :

"Luis, Pedro and Tomas earned 9 pesetas each, working at a store; how much did they earn in all?"

Javier, 7 years and 7 months (MA= 12 years and 2 months, IQ= 155), gave the correct answer in 5 seconds, adding that "I've got it in my mind because I had made it with my calculator".

Fernando, 11 years and 10 months (MA= 22 y. and 4 m.; IQ=177) solved right the above mentioned "jumper" problem before starting the chrono on.

Automation means the change taken from the conscious to subconscious state. That way, for example, when an individual learns driving, reading or speaking another language, he/she usually turns controlled information into automatical.

Abilities suitable for facing situations, solving novel problems or automating previous information are closely connected. And therefore, we can state that facing the novel leads to automation. At the same time, automation, as requiring a higher capacity, releases a greater amount of mental mechanism to deal with the novelty. So, the more an individual automates different aspects of a task or problem, the greater attention he/she will pay for the novel aspects of the given task. Each ability can potentially make operate easier another one. These two intelligence aspects functions in interaction with the Intelligence Triad Model components. Thus, the capacity for facing novel situations means applying the intelligence components to each individual's situation and problems. Automation isn't an isolated body functioning separately, but is related to intelligence components, taking actively part in information-processing.

Automation is reported to be the capacity to internalize everything learned. Its efficiency level depends on the following factors :

a - cohesion during information-processing

b - a proper performance of the process to be automated: automation aims to provide the appropiate procedure of learning, and mistakes are useful as a source of learning.

c - practice, which leads to strengthen what an individual learns.

d - attention focused on what one is doing.

e - thinking over all context where the task can be performed.

f - abstracting properly what is learned.

g - motivation (Prieto & Pérez, 1993, p. 65-66)

b - the gifted know which strategy they usually use for solving problems.

> **Juan**, 7 years and 7 months (MA=12 y. and 10 m.; IQ= 164), said: "I don't mind dividing, because I do just the contrary, multiplying; and the same when subtracting, I mentally turn it into addition".

This kind of statements made us research what kind of strategies gifted children use for solving maths problems, and its difference from average, retarded and older children.

We remember that a strategy means a general technique to solve problems and so, although it can't guarantee the solution, it is a guide to solve any problem. In this sense, Polya (1968) suggested, as an example of strategy, that the problem should be divided into sub-problems (Mayer, 1986; p. 429).

We proved that the children being from 6 to 8 could use at least 3 different strategies to solve maths problems :

a) operations with base 10

Example: A milkman has 25 milk bottles, if he sold 14, how many are there left?"
The operation made by a child who was 6 years and 7 months (MA= 10 years; IQ= 148) was the following : 25 - 10 = 15, 15 - 4 = 11, he answered right in 12 seconds.

b) simplifying before operating

Example : "if an orange cost is 9 pesetas, how much are 3 oranges?"
The child, 7 years and 6 months; MA= 12 y. IQ= 155, answered in 5 seconds: "I knew that 9+9 = 18; so, I only had to add 10 and subtract 1.

c) making task performance easier

Example : "if 3 sweets cost 5 pesetas, how much are 24 sweets?"
A child, 7 years and 7 months (MA= 12 years and 10 months, IQ = 164) answered right in

12 s.: he wondered what number multiplied by 3 had as result 24; once he found 8;
8 x 5 = 40.

Thus, it can be concluded that gifted children solve maths problems in a more efficient and special way than their peers; and that it isn't their knowledge but reasoning what let them solve problems, in view of their early age and no previous instruction concerning the operations involved.

As being their reasoning quality which guides gifted children's achievement, the efficient memory processes have a general sense for each information.

With regard to the strategy use, we found that the gifted, from an early age, can use efficient strategies for solving problems without having been taught. The way they solve maths tasks, differ from adult's, since the latter can use mathematical concepts as algorithm.

In relation with retarded children, the most important point of differing is that if the retarded are able to use the appropiate strategy when having been previously instructed; the gifted generate efficient strategies spontaneously, moreover, their strategy production for solving maths tasks occurs before, at an earlier age, and differs qualitatively from those which are produced by adults, the retarded or peers.

The present research results contributes to a deeper understanding of cognitive psychology and giftedness research, and they also are useful to provide strategies for curricular design. It is to be remembered Carpennter, Corbitt, Kepner, Lindquist & Reyes (1980) conclusions on the administration of some maths problems like this : "each 56c. bottle of limonade costed 95 pesetas, during last school fair, Roberto sold 8c. glasses at 20 pesetas each; how much did the school earn each bottle?". This kind of tasks were administrated to 70.000 students and only 11 % being 13 years plus 29 being 17 were able to solve right (Mayer 1986, p. 406). Skemp (1971, p. 13) pointed that both learning and teaching problems are psychological ones, and that knowing as much as possible how learning is made, is essential to improve maths teaching.

B · SOLVING COMPLEX PROBLEMS OF TRANSFORMATION

No reference on complex problem solving has been found in the giftedness field; it has only been suggested that this kind of problems, as "the problem of canivals and of missionaries", should be studied from the developmental point of view starting on secondary education (Sternberg, 1987, p 443-444).

The children observed through this research showed that they can solve complex problems from 7 years and with above 150 IQ, decreasing the high IQ required, when increasing children age.

In these sense, we propose the following examples to show how the gifted solve this kind of transformation problems, as Hanoi's Tower, which is usually solved by a sub-objective strategy (Wood 1987, p. 72).

According to Newell & Simon (1972), the most important general strategy for solving transformation problems is "medium-end analysis", what means, the "solver" compares continually each step with the objective, and selects any movement with regard to the differences arisen (Vega 1985, p. 499).

The starting point of Hanoi's problem consists of some rings (from 3 to 6) stringged at the stick /a/. The objective is placing all rings at the stick /c/, under these rules:
a - only one ring can be moved each time

b - no ring can be placed on another smaller than it. The stick /b/ can be used for intermediate movements.

Because of the advantages this problem offers to examiners, it has been widely used by researchers, like Simon (1975), Anzai & Simon (1979), Kadat (1982). This is a very complex task for any adult since it implies perceptive operations, complex strategies to make movements sequential, and an ample operative memory.

As above mentioned, children older than 7 are not only able to solve it with 5 rings, but also to infer the strategy used and to find out the precise strategy of perfect realization with as less movements as possible. Besides children were not given any corrective feedback to solve the task.

Raúl, for instance, who was 7 years and 10 months (MA= 13y and 10m; IQ=177) succeeded in Hanoi's Tower task during his attendance to MEPS program.

He found the precise way to do it perfectly and with the lest number of movements. As well as **Daniel**, 10 years and 2 months (MA=16y and 3m; IQ=147) and his brother **Jorge**, 11 years and 7 months (MA=18y 7m; IQ= 148) did.

Therefore, gifted capacity for generating an executive strategy must be underlined. This strategy consists of making up a procedure to solve a given kind of problems (Case, 1989, p. 94).

Francisco, 8 years and 8 months (MA=22y 0m, IQ=160), found out the number of movements required to solve the problem depending upon the given number of rings.

SEE ATTACHED PHOTOCOPY

So, according to Francisco, there exists a **mathematical rule** to solve this task depending on the number of rings.

Surprisingly, so early aged children can manage such a complex packet of information, lacking previous experience.

A research work on the relation between performing this task and the intelligence measured would be an essential understanding of individual differences. With regard to its application, certain aspects of this problem performing are resorted to solve regular life problems: determining intermediate aims, organizing the information, using some kind of evaluating resource to analyze any problem, are ways of solving real situations (Sternberg, 1987, p. 445).

The resulting data from this present research make us know better **cognitive psychology** and contribute to any research dealing with gifted children's ways of solving problems and the strategies used in relation to peers, average and retarded children. The most surprising data is the gifted students' capacity for infering executive strategies and elaborating the space of a complex problem.

Through this presentation we intended to show gifted children's strategies of problem solving, which can be useful to design curriculum planning; and to bring empirical data for the literature concerned with general aspects of cognition and intelligence, and the existing relations between reasoning and problem solving.

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